SYRIAN PRIVATE UNIVERSITY
الجامعة السورية الخاصة

كـلـيــة هندسة الحاسوب والمعلوماتية
Computer and Informatics Engineering Faculty

Electric Circuits I

Dr. Eng.
Hassan M. Ahmad

Hassan.Ahmad@spu.edu.sy, istamo48@mail.ru

Chapter 3
Methods of Analysis

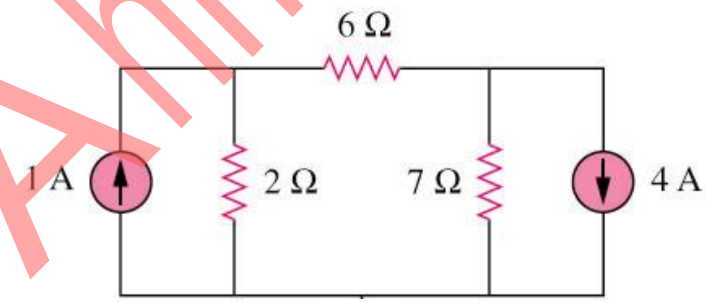
3.1 Motivation (الدافعية/التحفيز)

If you are given the following circuit, how can we determine:

(1) The voltage across each resistor,

(2) Current through each resistor,

(3) Power generated by each current source, etc.



What are the things which we need to know in order to determine the answers?

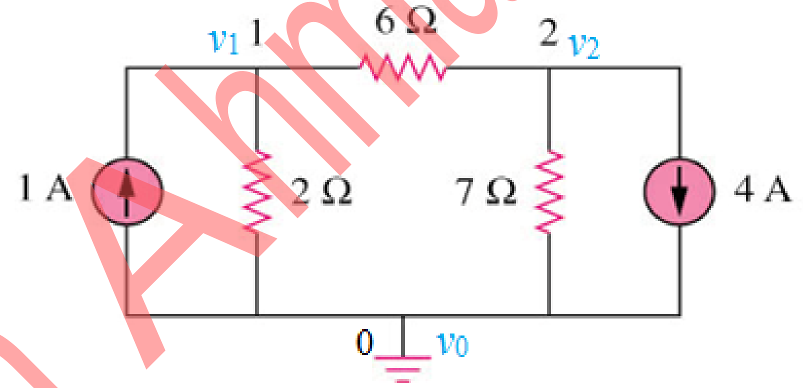
Things we need to know in solving any resistive circuit with current and voltage sources only:

- Kirchhoff's Current Laws (KCL);
- Kirchhoff's Voltage Laws (KVL)
- Ohm's Law

How should we apply these laws to determine the answers?

3.2 Nodal Analysis

It provides a general procedure for analyzing circuits using node voltages as the circuit variables.



Steps to determine the node voltages:

1. Select a node as the reference node, for example 0 (ground).
2. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Note: Current flows from a higher potential to a lower potential in a resistor.

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

Example 3.1.

Calculate the node voltages in the circuit shown in Fig.(a).

***Solution*:**

Consider Fig. (b).

- The reference node is selected (ground), and the node voltages and are now determined.
- The branch currents are determined.
- At node v_1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3 \Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2} \Rightarrow 3v_1 - v_2 = 20 \qquad (1)$$

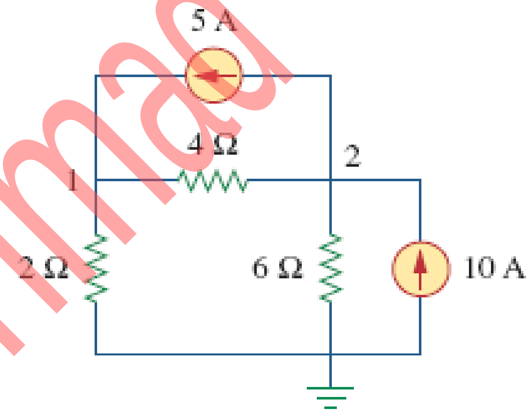
- At node v_2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

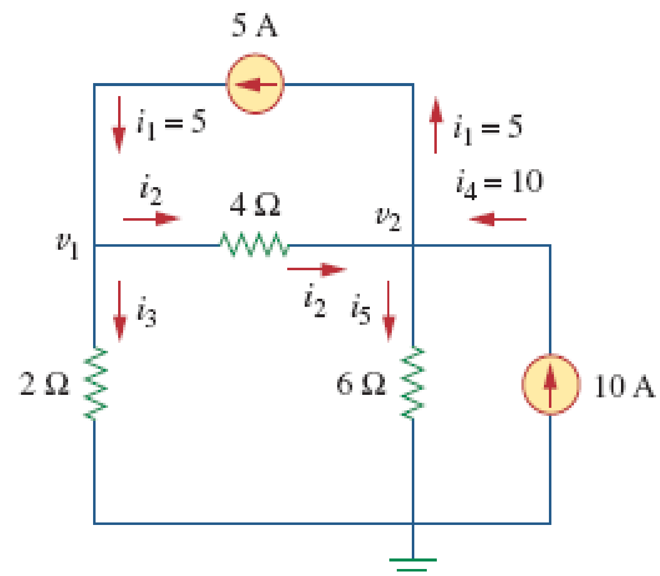
$$\Rightarrow -3v_1 + 5v_2 = 60 \qquad (2)$$

- Using the elimination (الإقصاء=الحذف والتعويض) or Cramer's rule (matrix form) techniques, gives:

$$v_1 = 13.333 \text{ V}; \qquad v_2 = 20 \text{ V}$$



(a)



(b)

Example 3.2.

Determine the voltages at the nodes in Fig.(a).

***Solution*:**

Consider Fig. (b).

At node 1,

$$3 = i_1 + i_x \Leftrightarrow 3 = \frac{v_1 - v_3}{4} + \frac{v_2 - v_2}{2}$$

$$\Rightarrow 3v_1 - 2v_2 - v_3 = 12 \qquad (1)$$

At node 2,

$$i_x = i_2 + i_3 \Leftrightarrow \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

$$\Rightarrow -4v_1 + 7v_2 - v_3 = 0 \qquad (2)$$

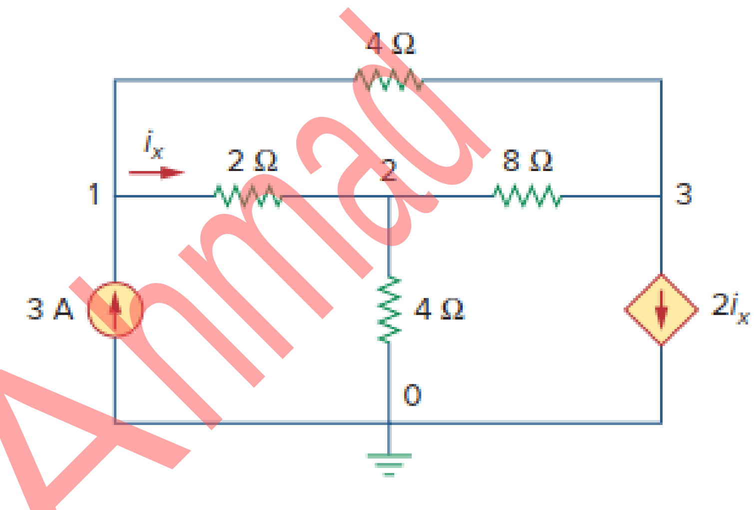
At node 3,

$$i_1 + i_2 = 2i_x \Leftrightarrow \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

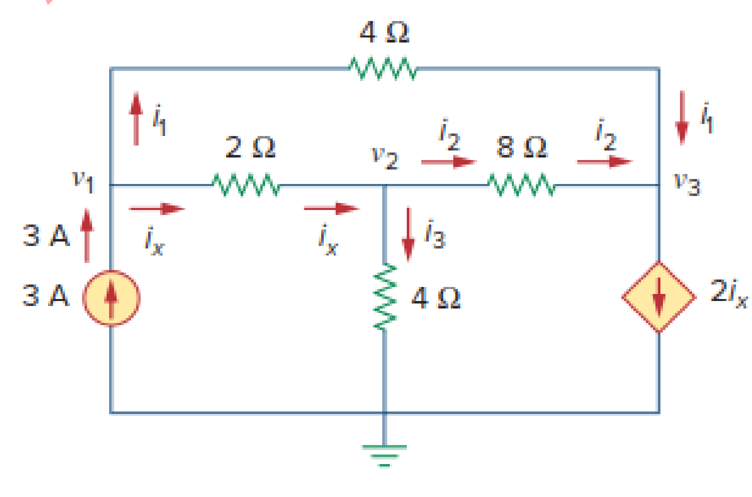
$$\Rightarrow 2v_1 - 3v_2 + v_3 = 0 \qquad (3)$$

By solving the eq. (1), (2) and (3) we get:

$$v_1 = 4.8\,\text{V}, \quad v_2 = 2.4\,\text{V}, \quad v_3 = -2.4\,\text{V}$$



(a)



(b)

3.3 Nodal Analysis with Voltage Source

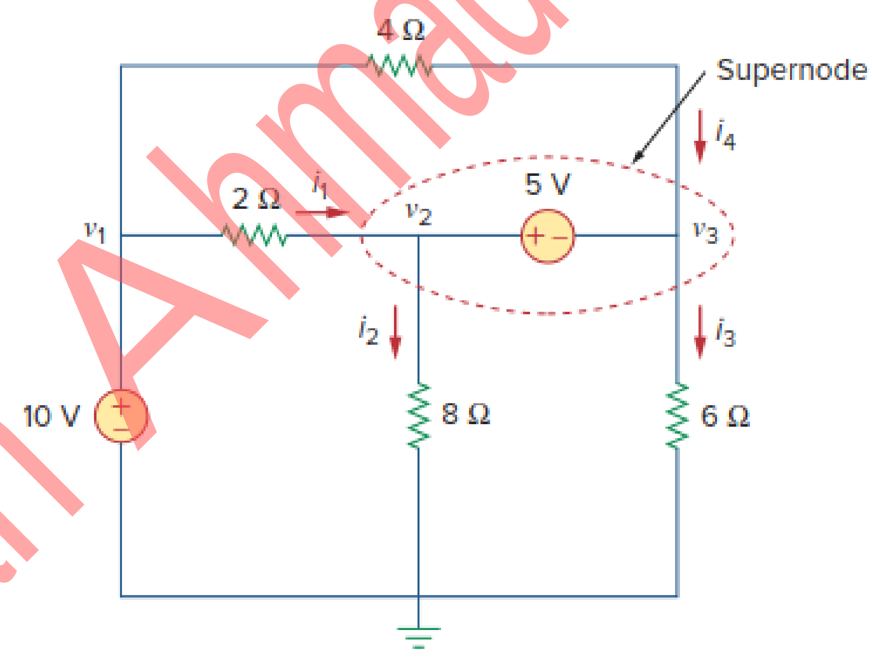
CASE 1. If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In Fig. , for example, $v_1 = 10\ \text{V}$

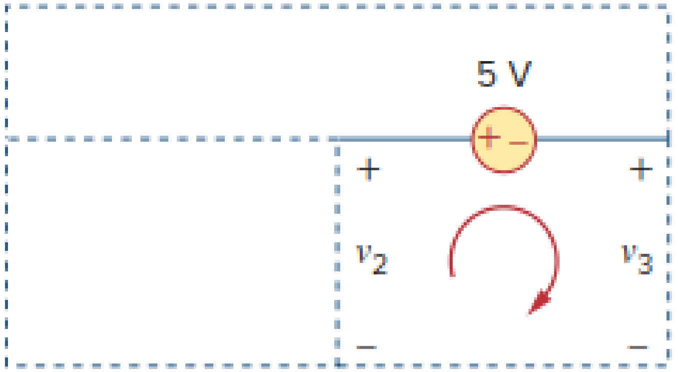
CASE 2. If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a **supernode**; we apply both KCL and KVL to determine the node voltages.

At the supernode, KCL gives:

$$i_1 + i_4 = i_2 + i_3$$

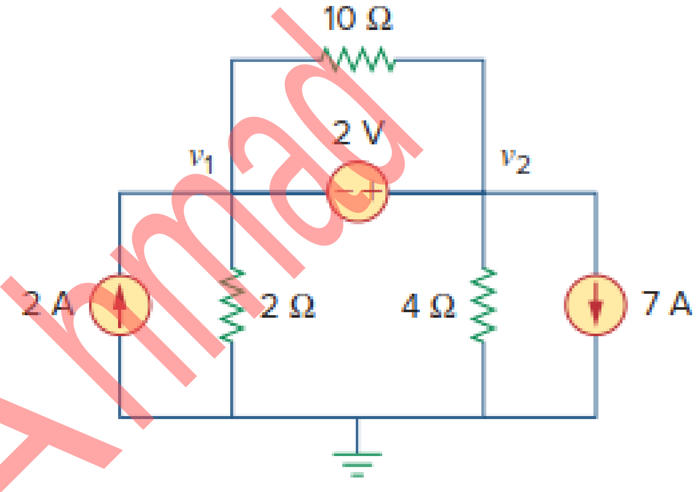
and KVL gives $-v_2 + 5 + v_3 = 0$





Example 3.3

For the circuit shown in Fig., find the node voltages.

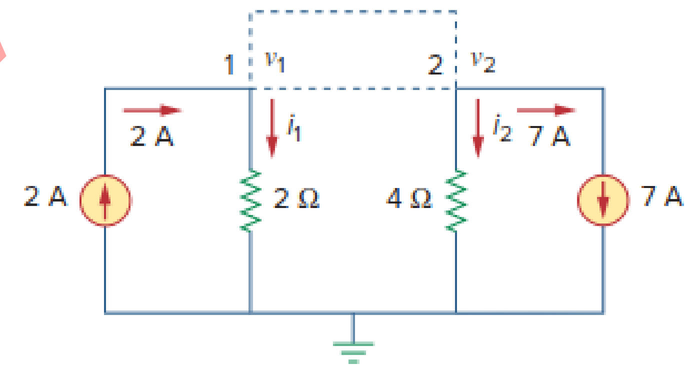


Solution

Applying KCL to the supernode as shown in Fig.(a) gives

$$2 = i_1 + i_2 + 7 \Leftrightarrow 2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7$$

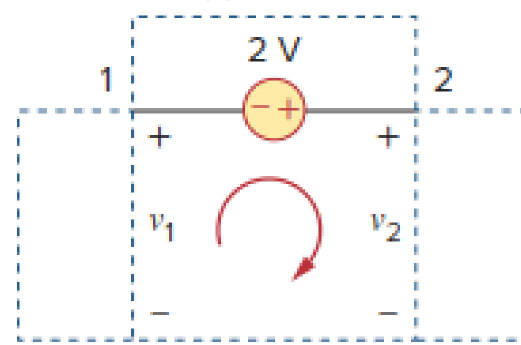
$$\Rightarrow v_2 = -20 - 2v_1 \qquad (1)$$



(a)

To get the relationship between v_1 and v_2, we apply KVL to the circuit in Fig.(b).

$$-v_1 - 2 + v_2 = 0 \Rightarrow v_2 = v_1 + 2 \qquad (2)$$



(b)

From Eqs. (1) and (2), we write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

$$\Rightarrow v_1 = -7.333 \text{ V}, \quad v_2 = -5.333 \text{ V}$$

Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.

Example 3.4

Find the node voltages in the circuit of Fig.

Solution

We apply KCL to the two supernodes as in Fig. (a).

At supernode 1-2,

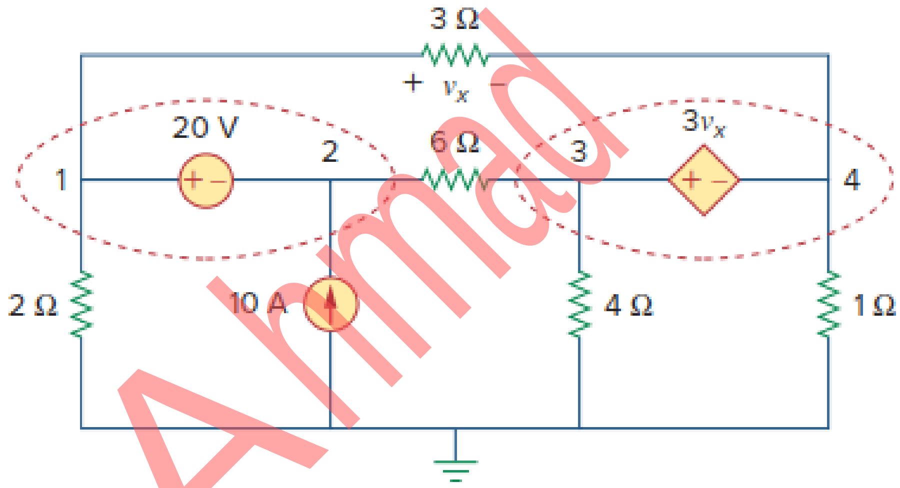
$$i_3 + 10 = i_1 + i_2 \Leftrightarrow \frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

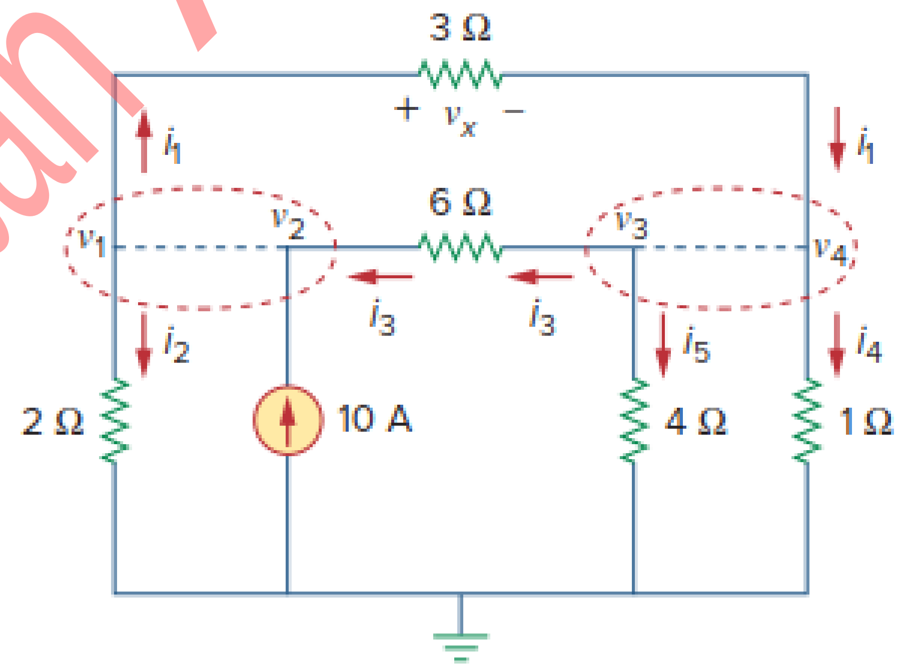
$$\Rightarrow 5v_1 + v_2 - v_3 - 2v_4 = 60 \quad (1)$$

At supernode 3-4,

$$i_1 = i_3 + i_4 + i_5 \Leftrightarrow \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

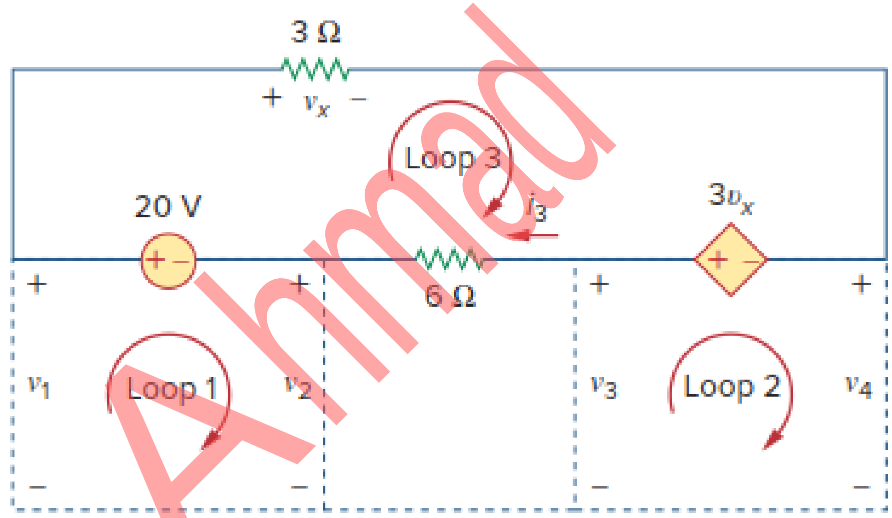
$$\Rightarrow 4v_1 + 2v_2 - 5v_3 - 16v_4 = 0 \quad (2)$$





(a)

We now apply KVL to the branches involving the voltage sources as shown in Fig.(b).



(b)

For loop 1,

$$-v_1 + 20 + v_2 = 0 \Rightarrow v_1 - v_2 = 20 \qquad (3)$$

For loop 2,

$$-v_3 + 3v_x + v_4 = 0, \quad \text{but } v_x = v_1 - v_4$$

$$\Rightarrow 3v_1 - v_3 - 2v_4 = 0 \qquad (4)$$

For loop 3, $v_x - 3v_x + 6i_3 - 20 = 0, \quad \text{but } 6i_3 = v_3 - v_2 \text{ and } v_x = v_1 - v_4$

Hence, $-2v_1 - v_2 + v_3 + 2v_4 = 20 \qquad (5)$

Solve the Eqs. (1), (2), (3), (4) and (5) we get:

$$v_1 = 26.67 \text{ V}, \quad v_2 = 6.667 \text{ V}$$

$$v_3 = 173.33 \text{ V}, \quad v_4 = -46.67 \text{ V}$$

3.4 Mesh Analysis

1. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.
2. Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.
3. A mesh is a loop which does not contain any other loops within it.

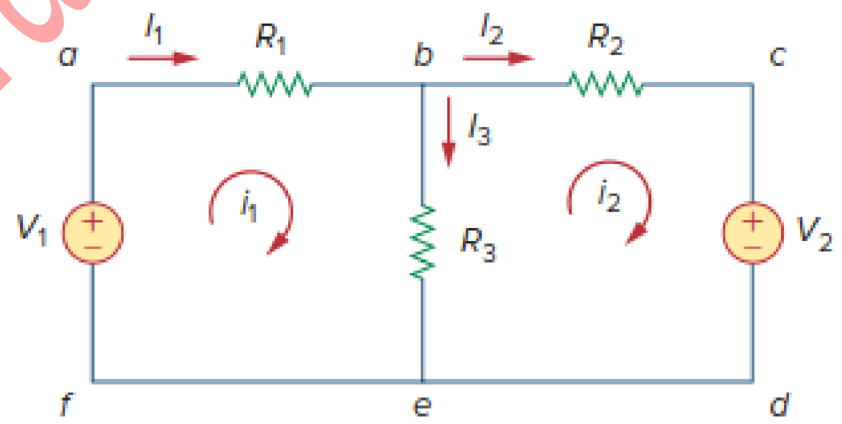
Steps to determine the mesh currents:

1. Assign mesh currents i_1, i_2, …, in to the n meshes.
2. Apply KCL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

NOTE.

For mesh 1: $I_3 = i_1 - i_2$

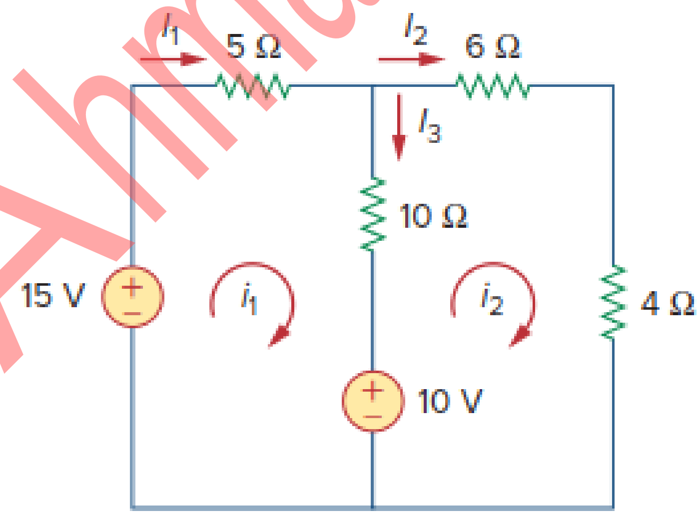
For mesh 2: $I_3 = i_2 - i_1$



Example 3.5

For the circuit in Fig., find the branch currents I_1, I_2, and I_3 using mesh analysis.

Solution



For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0 \Rightarrow 3i_1 - 2i_2 = 1 \quad (1)$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0 \Rightarrow i_1 = 2i_2 - 1 \quad (2)$$

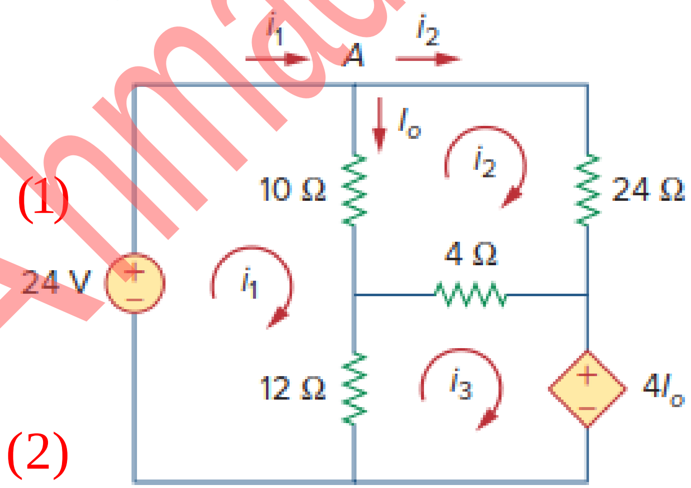
By solving the Eqs. (1) and (2) we get:

$$i_1 = 1\text{A}, \quad i_2 = 1\text{A} \Rightarrow$$

$$I_1 = i_1 = 1\text{A}, \quad I_2 = i_2 = 1\text{A}, \quad I_3 = i_1 - i_2 = 0\,\text{A}$$

Example 3.6

Use mesh analysis to find the current I_o in the circuit of Fig.



Solution

For mesh 1,

$$-24+10(i_1-i_2)+12(i_1-i_3)=0 \Rightarrow 11i_1-5i_2-6i_3=12 \quad (1)$$

For mesh 2,

$$24i_2+4(i_2-i_3)+10(i_2-i_1)=0 \Rightarrow -5i_1+19i_2-2i_3=0 \quad (2)$$

For mesh 3, $4I_o+12(i_3-i_1)+4(i_3-i_2)=0,\quad \text{but } I_o=i_1-i_2$

$$4(i_1-i_2)+12(i_3-i_1)+4(i_3-i_2)=0 \Rightarrow -i_1-i_2+2i_3=0 \quad (3)$$

By solving the Eqs. (1) and (2) we get: $i_1=2.25\,\text{A},\quad i_2=0.75\,\text{A},\quad i_3=1.5\,\text{A}$

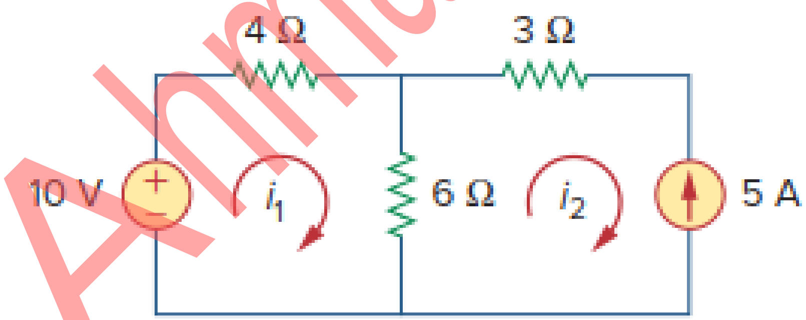
$$\Rightarrow I_o=i_1-i_2=1.5\,\text{A}$$

3.5 Mesh Analysis with Current Source

CASE 1.

When a current source exists only in one mesh.

Consider the circuit in Fig.



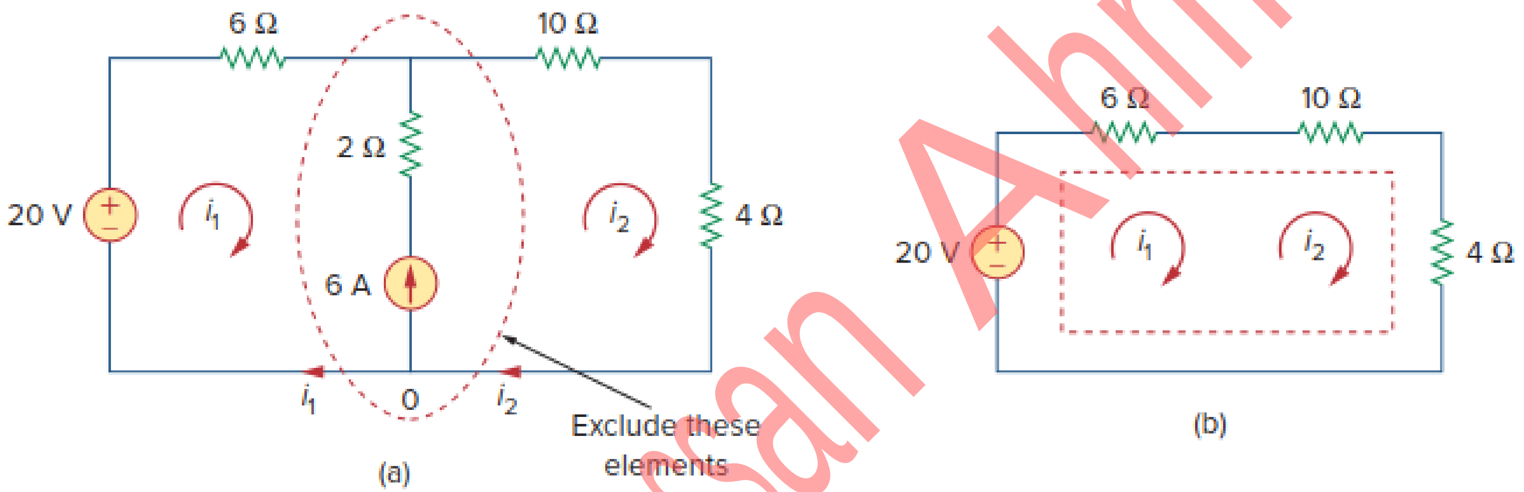
We set $i_2 = -5$ and write a mesh equation for the other mesh in the usual way; that is,

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \Rightarrow i_1 = -2\text{A}$$

CASE 2.

When a current source exists between two meshes, a **super-mesh** is resulting.

Consider the circuit in Fig.(a).



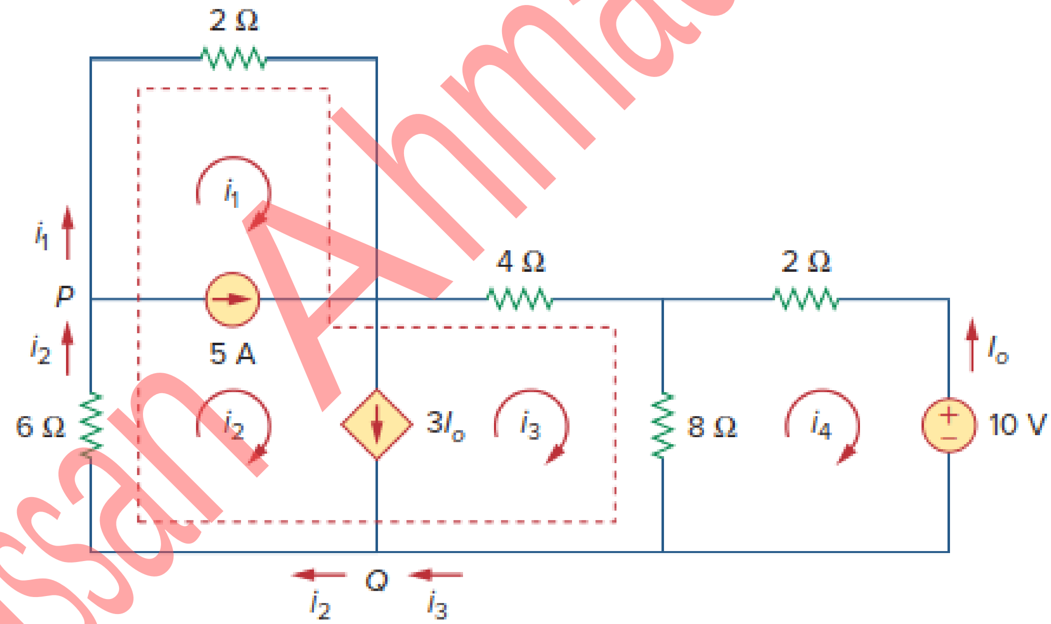
We create a *supermesh* by excluding the current source and any elements connected in series with it, as shown in Fig.(b). Thus,

- Applying KVL to the supermesh in Fig.(b) gives $-20+6i_1+10i_2+4i_2=0$ (1)
- Applying KCL to a node in the branch where the two meshes intersect (node 0) in Fig (a) gives $i_2=i_1+6$ (2)

By solving Eqs. (1) and (2), we get $i_1=-3.2\ \text{A}, \quad i_2=2.8\ \text{A}$

Example 3.7

For the circuit in Fig., find i_1 to i_4 using mesh analysis.



Solution

Apply KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$\Rightarrow i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \qquad (1)$$

Apply KCL to node *P*:

$$i_2 = i_1 + 5 \qquad (2)$$

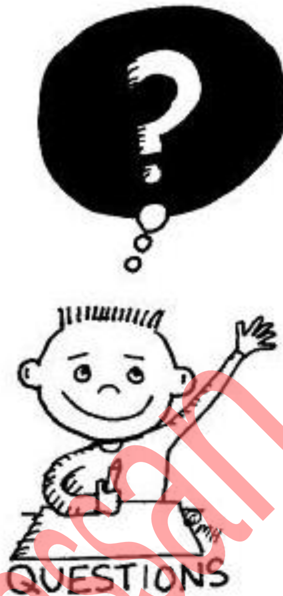
Apply KCL to node *Q*:

$$i_2 = i_3 + 3I_o, \quad \text{but} \quad I_o = -i_4 \Rightarrow i_2 = i_3 - 3i_4 \qquad (3)$$

Apply KVL in mesh 4, $2i_4 + 8(i_4 - i_3) + 10 = 0 \Rightarrow 5i_4 - 4i_3 = -5 \qquad (4)$

By solving Eqs. (1) … (4), we get

$$i_1 = -7.5\,\text{A}, \quad i_2 = -2.5\,\text{A}, \quad i_3 = 3.93\,\text{A}, \quad i_4 = 2.143\,\text{A}$$



The end of chapter 3